Useful Models for Robot Learning

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ELLIS Workshop

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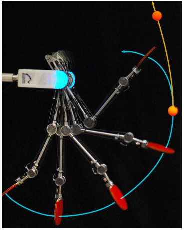
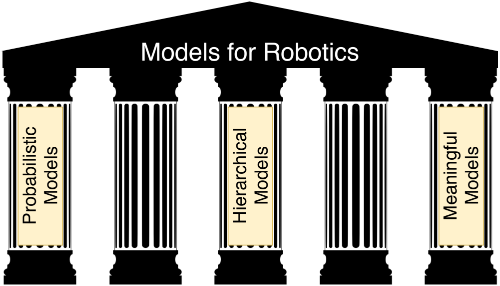
Challenges in Robot Learning

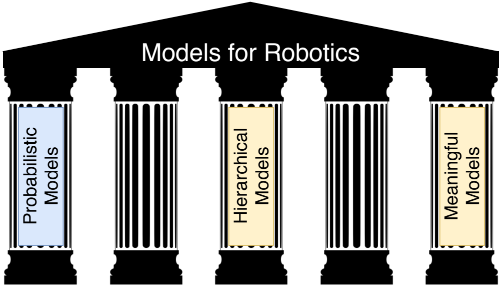




- Automatic adaption in robotics ⏩ Learning
- Practical constraint: data efficiency
- Models are useful for data-efficient learning in robotics

3 Models for Data-Efficient Robot Learning



1. **Probabilistic models**
   - Fast reinforcement learning
2. **Hierarchical models**
   - Infer task similarities within a meta-learning framework
3. **Physically meaningful models**
   - Encode real-world constraints into learning



Carl Rasmussen

Dieter Fox

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t, \boldsymbol{u}_t) + \boldsymbol{w}\,, \qquad \boldsymbol{u}_t = \pi(\boldsymbol{x}_t, \boldsymbol{\theta})$$

State

Transition function

Control

Policy

Policy parameters

$$x_{t+1} = f(x_t, u_t) + w\,, \qquad u_t = \pi(x_t, \theta)$$

State — Transition function — Control — Policy — Policy parameters

Objective (Controller Learning)

Find policy parameters θ^* that minimize the expected long-term cost

$$J(\theta) = \sum_{t=1}^{T} \mathbb{E}[c(x_t)|\theta]\,, \qquad p(x_0) = \mathcal{N}(\mu_0, \Sigma_0)\,.$$

Instantaneous cost $c(x_t)$, e.g., $\|x_t - x_{\text{target}}\|^2$

- Typical objective in optimal control and reinforcement learning (Bertsekas, 2005; Sutton & Barto, 1998)

Fast Reinforcement Learning

Objective

Minimize expected long-term cost $J(\boldsymbol{\theta}) = \sum_t \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]$

Deisenroth et al. (IEEE-TPAMI, 2015): *Gaussian Processes for Data-Efficient Learning in Robotics and Control*

Objective

Minimize expected long-term cost $J(\boldsymbol{\theta}) = \sum_t \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]$

PILCO Framework: High-Level Steps

1. Probabilistic model for transition function
   - ▸▸ System identification

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2. Compute long-term state evolution $p(\boldsymbol{x}_1|\boldsymbol{\theta}), \ldots, p(\boldsymbol{x}_T|\boldsymbol{\theta})$
3. Policy improvement
4. Apply controller

Deisenroth et al. (IEEE-TPAMI, 2015): *Gaussian Processes for Data-Efficient Learning in Robotics and Control*

Objective

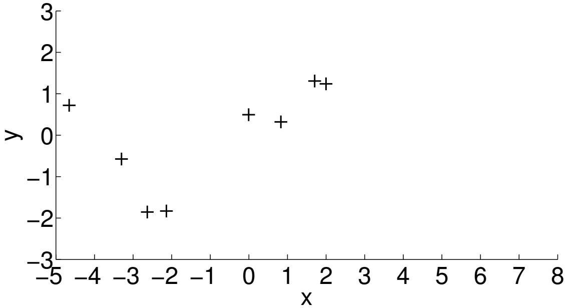
Minimize expected long-term cost $J(\boldsymbol{\theta}) = \sum_t \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]$

PILCO Framework: High-Level Steps

1. **Probabilistic model for transition function f**
   ▸▸ **System identification**
2. Compute long-term predictions $p(\boldsymbol{x}_1|\boldsymbol{\theta}), \ldots, p(\boldsymbol{x}_T|\boldsymbol{\theta})$
3. Policy improvement
4. Apply controller

Deisenroth et al. (IEEE-TPAMI, 2015): *Gaussian Processes for Data-Efficient Learning in Robotics and Control*

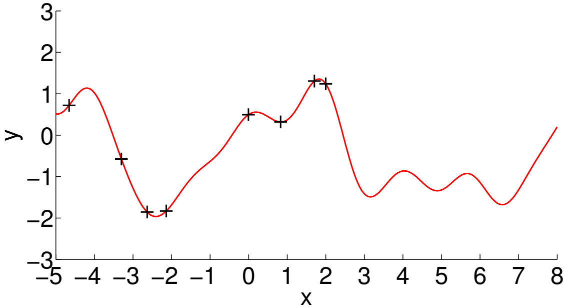
Model learning problem: Find a function $f : x \mapsto f(x) = y$



Observed function values

Model Learning (System Identification)

Model learning problem: Find a function $f : x \mapsto f(x) = y$



Plausible model

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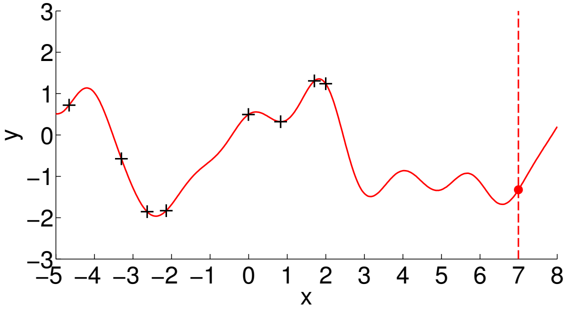

Plausible model

Predictions? Decision Making?

Model Learning (System Identification)

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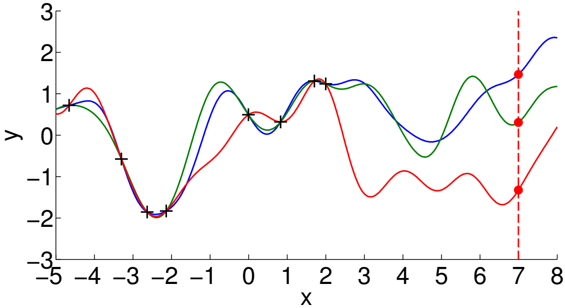
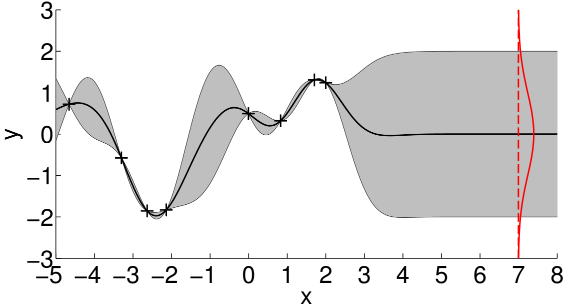


More plausible models

Predictions? Decision Making? Model Errors!

Model Learning (System Identification)

Model learning problem: Find a function $f : x \mapsto f(x) = y$



Distribution over plausible functions

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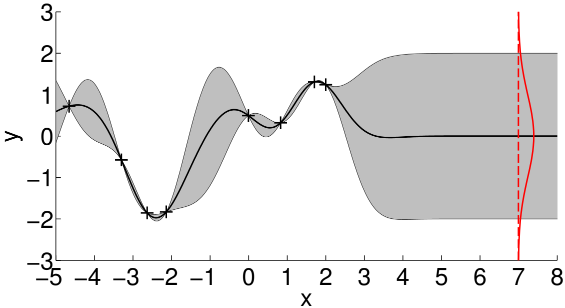


Distribution over plausible functions

- Express uncertainty about the underlying function to be robust to model errors
- Gaussian process for model learning (Rasmussen & Williams, 2006)

Objective

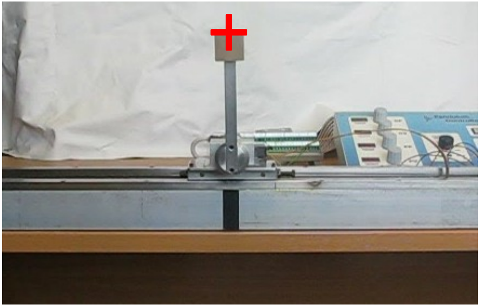
Minimize expected long-term cost $J(\boldsymbol{\theta}) = \sum_t \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]$

PILCO Framework: High-Level Steps

1. Probabilistic model for transition function f
   ▸▸ System identification
2. Compute long-term predictions $p(\boldsymbol{x}_1|\boldsymbol{\theta}), \ldots, p(\boldsymbol{x}_T|\boldsymbol{\theta})$
3. Policy optimization via gradient descent
4. Apply controller

Deisenroth et al. (IEEE-TPAMI, 2015): *Gaussian Processes for Data-Efficient Learning in Robotics and Control*

Standard Benchmark: Cart-Pole Swing-up



- Swing up and balance a freely swinging pendulum on a cart
- No knowledge about nonlinear dynamics ⏩ Learn from scratch
- Cost function $c(\boldsymbol{x}) = 1 - \exp(-\|\boldsymbol{x} - \boldsymbol{x}_{\text{target}}\|^2)$

- Code: https://github.com/ICL-SML/pilco-matlab

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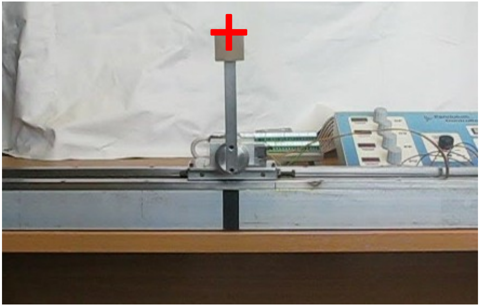

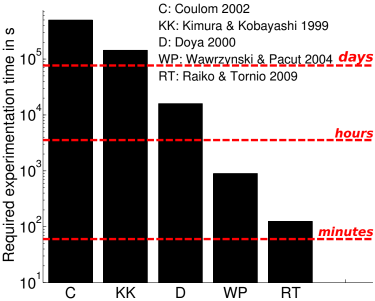

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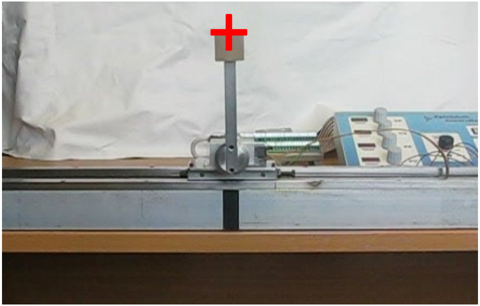
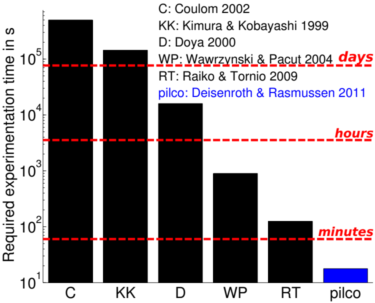
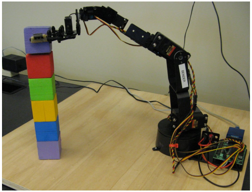




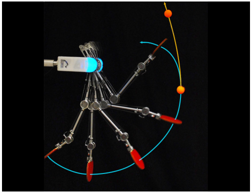
- Swing up and balance a freely swinging pendulum on a cart
- No knowledge about nonlinear dynamics ⏩ Learn from scratch
- Cost function $c(\boldsymbol{x}) = 1 - \exp(-\|\boldsymbol{x} - \boldsymbol{x}_{\text{target}}\|^2)$
- **Unprecedented learning speed** compared to state-of-the-art
- Code: https://github.com/ICL-SML/pilco-matlab

Deisenroth & Rasmussen (ICML, 2011): *PILCO: A Model-based and Data-efficient Approach to Policy Search*

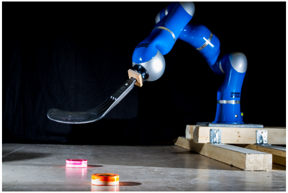
Wide Applicability



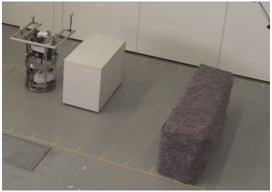
with D Fox



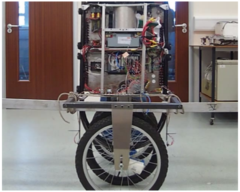
with P Englert, A Paraschos, J Peters



with A Kupcsik, J Peters, G Neumann



B Bischoff (Bosch), ESANN 2013



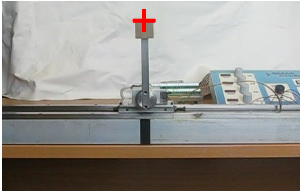
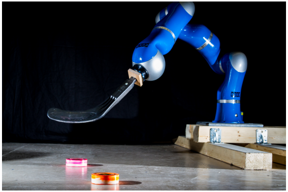
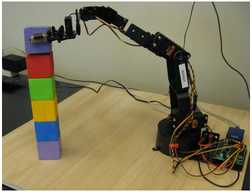
A McHutchon (U Cambridge)

B Bischoff (Bosch), ECML 2013

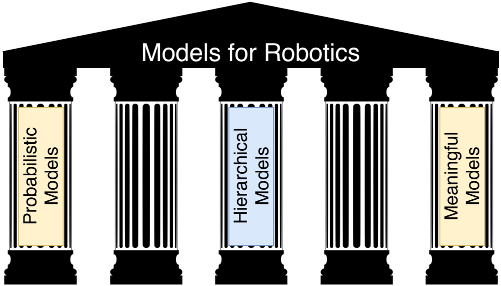
➤ Application to a wide range of robotic systems

Deisenroth et al. (RSS, 2011): *Learning to Control a Low-Cost Manipulator using Data-efficient Reinforcement Learning*
Englert et al. (ICRA, 2013): *Model-based Imitation Learning by Probabilistic Trajectory Matching*
Deisenroth et al. (ICRA, 2014): *Multi-Task Policy Search for Robotics*
Kupcsik et al. (AIJ, 2017): *Model-based Contextual Policy Search for Data-Efficient Generalization of Robot Skills*

Summary (1)

- In robotics, data-efficient learning is critical
- Probabilistic, model-based RL approach
    - Reduce model bias
    - Unprecedented learning speed
    - Wide applicability



Steindór Sæmundsson

Katja Hofmann

Meta Learning (Schmidhuber 1987)

Generalize knowledge from known tasks to new (related) tasks

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Generalize knowledge from known tasks to new (related) tasks

- Different robot configurations (link lengths, weights, ...)
- Re-use experience gathered so far generalize learning to new dynamics that are similar
  ▶▶ Accelerated learning

Approach

- Separate global and task-specific properties
- Shared global parameters describe general dynamics
- Describe task-specific (local) properties with latent variable

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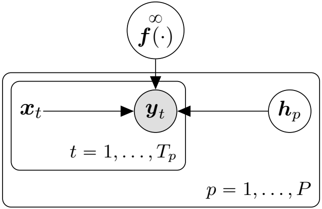
- Separate global and task-specific properties
- Shared global parameters describe general dynamics
- Describe task-specific (local) properties with latent variable
- Online variational inference of local properties

Meta Model Learning with Latent Variables

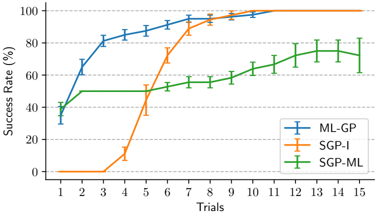


$$\boldsymbol{y}_t = \boldsymbol{f}(\boldsymbol{x}_t, \boldsymbol{h}_p)$$

- GP captures global properties of the dynamics
- Latent variable $\boldsymbol{h}_p$ encodes local properties
  - Variational inference to find a posterior on latent task

Sæmundsson et al. (UAI, 2018): *Meta Reinforcement Learning with Latent Variable Gaussian Processes*

Meta-RL (Cart Pole): Few-Shot Generalization

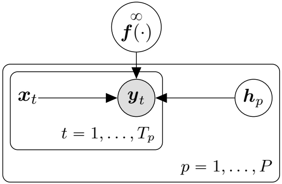


- Train on 6 tasks with different configurations (length/mass)
- Few-shot generalization on 4 unseen configurations
- Success: solve all 10 (6 training + 4 test) tasks
- Meta learning: blue
- Independent (GP-MPC): orange
- Aggregated experience model (no latents): green

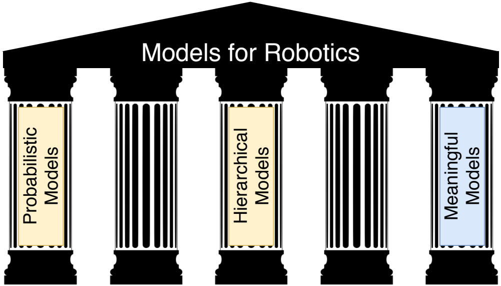
Meta RL generalizes well to unseen tasks

Sæmundsson et al. (UAI, 2018): *Meta Reinforcement Learning with Latent Variable Gaussian Processes*

Summary (2)



- Generalize knowledge from known situations to unseen ones
  ▶▶ **Few-shot learning**
- Latent variable can be used to infer task similarities
- Significant speed-up in model learning and model-based RL

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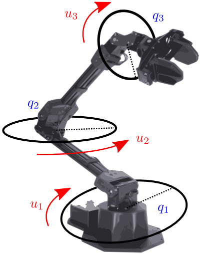
Alexander Terenin


Katja Hofmann

Physically Meaningful Models



Equations of motion

$$u = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q}$$

- Goal: Data efficiency and interpretability
- Inductive biases to account for physical/mechanical properties (e.g., conservation laws, configuration constraints)
  ▶▶ Learn dynamical systems that are "meaningful"

Neural Networks as Dynamical Systems

Approach:

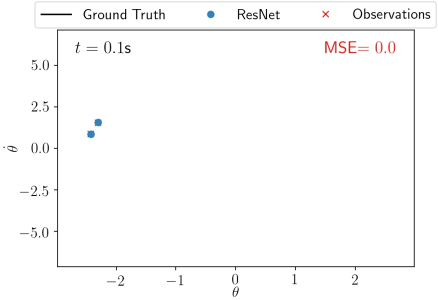
- Euler discretiztion of continuous-time dynamical system

$$\boldsymbol{x}(T)|\boldsymbol{x}_0 = \int_{t=0}^{T} f_\theta(\boldsymbol{x}(t))\mathrm{d}t \approx \boldsymbol{x}_0 + h\sum_{t=0}^{T-1} f_\theta(\boldsymbol{x}_t, t)$$

- Deep residual network
  (E, 2017; Haber & Ruthotto, 2017; Chen et al., 2018)

Example: Pendulum

- ODE: $\ddot{\theta} = -\frac{g}{l}\sin\theta$
- Observation: $\boldsymbol{y} = [\theta, \dot{\theta}]^\top$
- Training data: 15 seconds (150 data points)



- ODE: $\ddot{\theta} = -\frac{g}{l}\sin\theta$
- Observation: $\boldsymbol{y} = [\theta, \dot{\theta}]^\top + \boldsymbol{\epsilon}, \quad \epsilon \sim \mathcal{N}(\boldsymbol{0}, 0.33^2\boldsymbol{I})$
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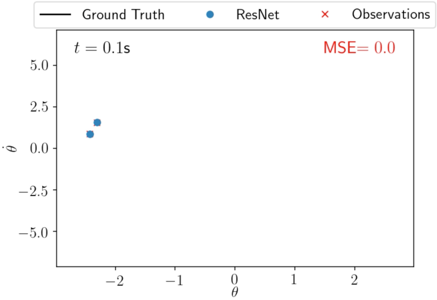

Example: Pendulum with Noisy Observations

- ODE: $\ddot{\theta} = -\frac{g}{l} \sin \theta$
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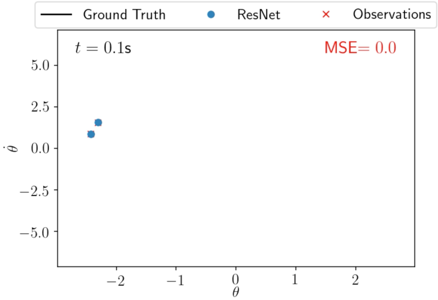
- Training data: 15 seconds



- Low prediction quality
- Does not obey physics
- ResNet does not conserve energy

Building Physics into Network Structure

- **Lagrangian**: Encodes "type" of physics, symmetries.

$$L(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t))$$

Building Physics into Network Structure

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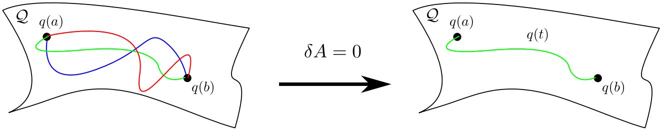
- **Hamilton's Principle**:

$$A = \int_a^b L(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t))\mathrm{d}t\,, \qquad \frac{\delta A}{\delta \boldsymbol{q}(t)} = \boldsymbol{0}$$



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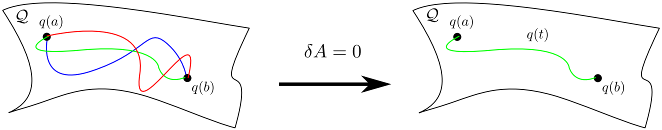
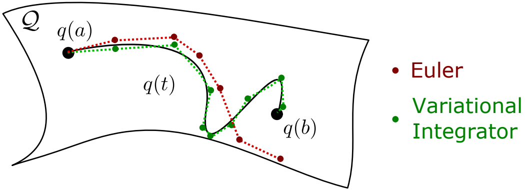


First idea:

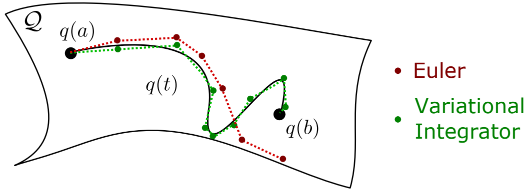
- Learn Lagrangian L instead of dynamics
- Encode physical properties via L (e.g., Lutter et al., 2019; Greydanus et al., 2019)

Variational Integrators



Second idea: Discretize in a way that preserves the physics

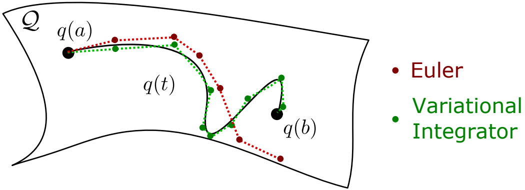
Variational Integrators



Second idea: Discretize in a way that preserves the physics

- Conservative, separable Newtonian system:

$$L_\theta(\boldsymbol{q}, \dot{\boldsymbol{q}}) = T_\theta(\dot{\boldsymbol{q}}) - U_\theta(\boldsymbol{q}) = \underbrace{\tfrac{1}{2}\dot{\boldsymbol{q}}^\top \boldsymbol{M}_\theta \dot{\boldsymbol{q}}}_{\text{kinetic}} - \underbrace{U_\theta(\boldsymbol{q})}_{\text{potential}}$$



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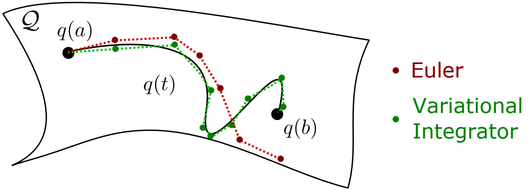
- Discretize action integral A

Variational Integrators



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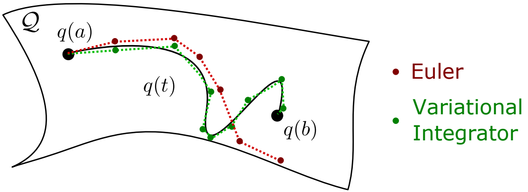
- Discretize action integral A
- Explicit variational integrator

$$\boldsymbol{x}_{t+1} = f_\theta(\boldsymbol{x}_1, t, h)\,, \quad \boldsymbol{x}_t := [\boldsymbol{q}_t, \boldsymbol{q}_{t-1}]$$

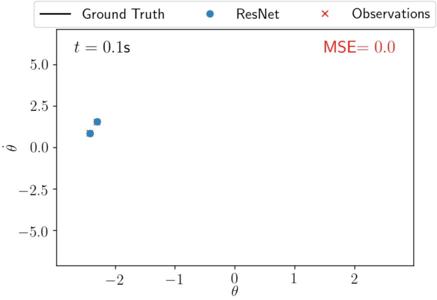
with initial condition $\boldsymbol{x}_1$

Variational Integrators: Properties



- Physical properties (e.g., conservation laws) automatically enforced
- Flexibility retained to model U_θ (e.g., with a neural network)
- Notions of kinetic and potential energy
  ▶▶ Increased interpretability

Example: Pendulum with Noisy Observations



Sæmundsson et al. (AISTATS 2020): *Variational Integrator Networks for Physically Structured Embeddings*

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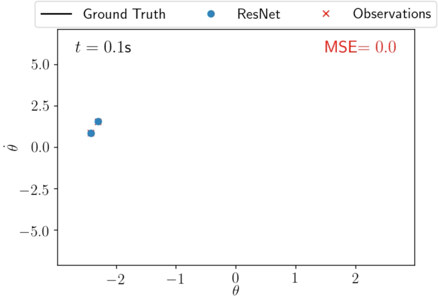

- Good predictive performance
- Obeys physics
- Conserves energy

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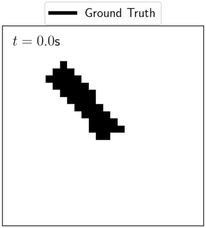
Learning from Pixels



Setting:

- Observations: 28×28 pixel images
- Training data: 60 images (6 seconds of pendulum movement)

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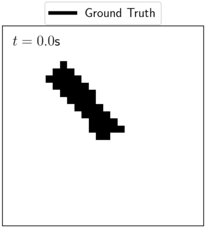
Learning from Pixels



Setting:

- Observations: 28×28 pixel images
- Training data: 60 images (6 seconds of pendulum movement)

Approach:

- Variational auto-encoder to embed pixels in low-dimensional space
- VIN within low-dimensional space

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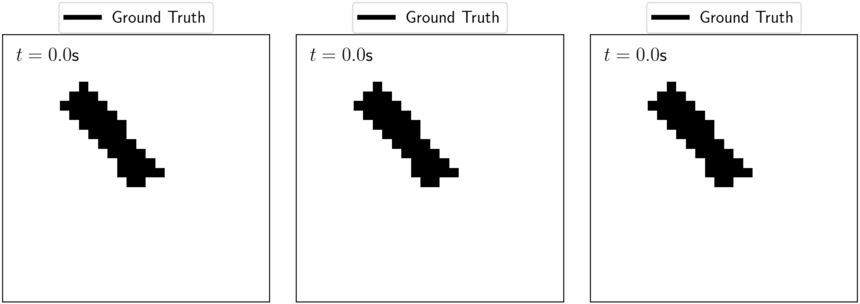
Results



- Residual RNN
- VIN
- VIN on SO(2)
- Code: `https://tinyurl.com/yx3yhhvo`

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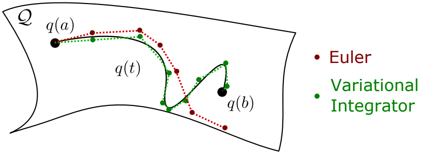
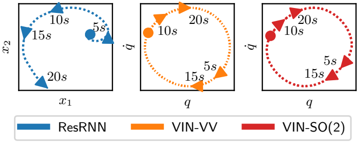
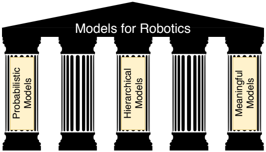
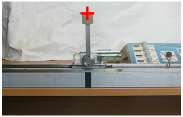


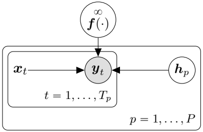


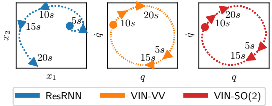
- Encode physics constraints when learning predictive models
- Variational integrator instead of Euler discretization
- Can be combined with VAE to learn predictive models from image observations
- Data efficient and interpretable

Wrap-up



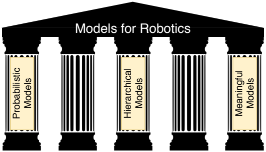


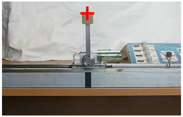


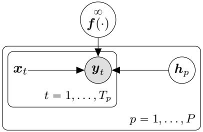


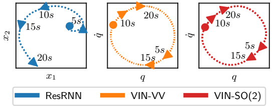
- **Data efficiency** is a practical challenge for autonomous robots
- Three useful models for data-efficient learning in robotics
  1. Probabilistic models for fast reinforcement learning
  2. Hierarchical models for learning task similarities within a meta-learning framework
  3. Physically meaningful models to encode real-world constraints into learning

Wrap-up









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Thank you for your attention

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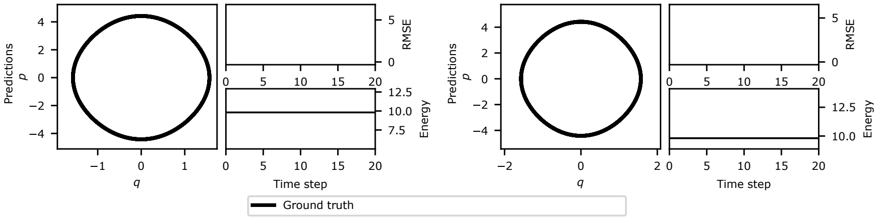
Learning from Noisy Data: Pendulum



Pendulum System. **Left:** 150 observations; **Right:** 750 observations.

Sæmundsson et al. (arXiv:1910.09349): *Variational Integrator Networks for Physically Meaningful Embeddings*

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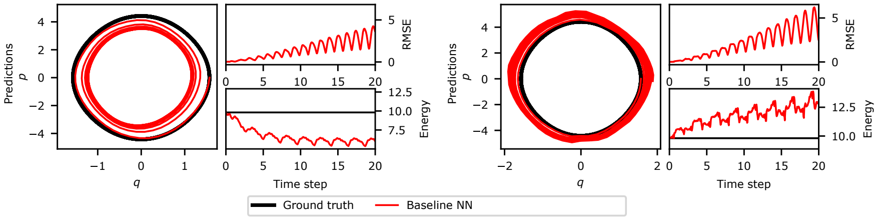

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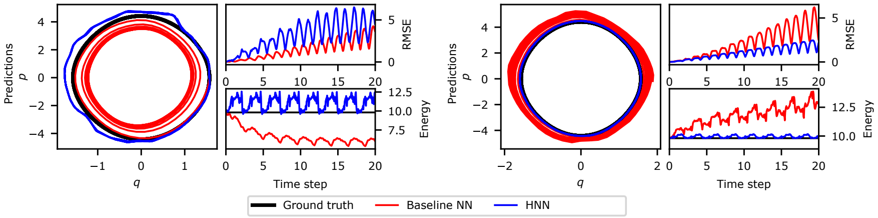

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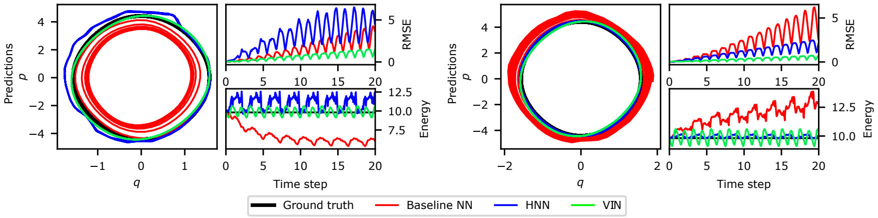

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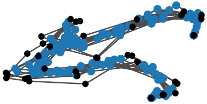
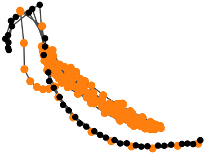
- Hamiltonian neural network (Greydanus et al., 2019): Overfits in low-data regime
- Variational integrator network: Conserves energy and generalizes better in both regimes

Sæmundsson et al. (arXiv:1910.09349): *Variational Integrator Networks for Physically Meaningful Embeddings*

Latent Embeddings of Time Series

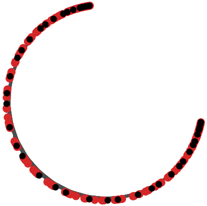


(a) VAE

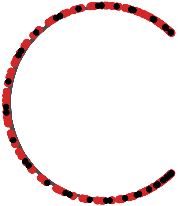


(b) Dynamic VAE

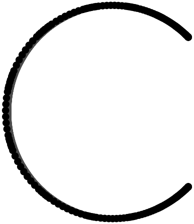
(c) Lie Group VAE



(d) VIN-$SO(2)$



(e) VIN-$SO(2)$ with fixed $\boldsymbol{M}$



(f) Ground Truth