# Tackling the Data-Efficiency Challenge in Autonomous Robots Using Probabilistic Modeling

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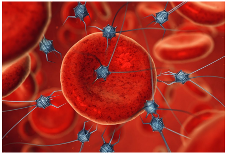
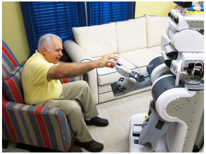
# Autonomous Robots

- Vision: Autonomous robots support humans in everyday activities ⏩ Fast learning and automatic adaptation

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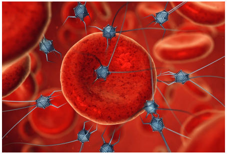  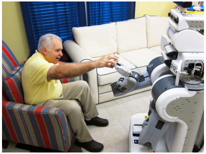 

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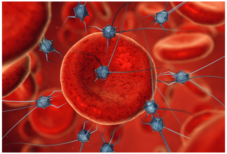
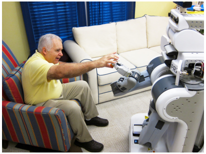




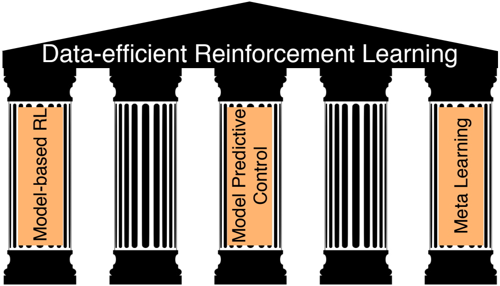

- Vision: Autonomous robots support humans in everyday activities ▶▶ Fast learning and automatic adaptation
- Currently: Data-hungry learning or human guidance

Fully **autonomous learning and decision making with little data** in real-life situations

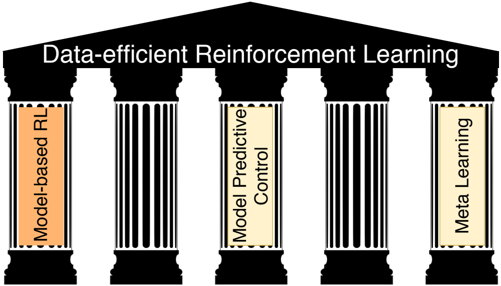
# Central Problem

**Data-Efficient Reinforcement Learning**

Ability to learn and make decisions in complex domains without requiring large quantities of data




1. **Model-based RL**
   - Data-efficient decision making
2. **Model predictive RL**
   - Speed up learning further by online planning
3. **Meta learning**
   - Generalization of knowledge to new situations


Data-efficient Reinforcement Learning
Model-based RL
Model Predictive Control
Meta Learning

# Reinforcement Learning

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t, \boldsymbol{u}_t) + \boldsymbol{w}\,, \qquad \boldsymbol{u}_t = \pi(\boldsymbol{x}_t, \boldsymbol{\theta})$$

State

Transition function

Control

Policy

Policy parameters

# Reinforcement Learning

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t, \boldsymbol{u}_t) + \boldsymbol{w}\,, \qquad \boldsymbol{u}_t = \pi(\boldsymbol{x}_t, \boldsymbol{\theta})$$

State  Control  Policy  Policy parameters

Transition function

## Objective (Controller Learning)

Find policy parameters $\boldsymbol{\theta}^*$ that minimize the expected long-term cost

$$J(\boldsymbol{\theta}) = \sum\nolimits_{t=1}^{T} \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]\,, \qquad p(\boldsymbol{x}_0) = \mathcal{N}\big(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0\big)\,.$$

Instantaneous cost $c(\boldsymbol{x}_t)$,  e.g., $\|\boldsymbol{x}_t - \boldsymbol{x}_{\text{target}}\|^2$

- Typical objective in optimal control and reinforcement learning (Bertsekas, 2005; Sutton & Barto, 1998)

# Fast Reinforcement Learning

## Objective

Minimize expected long-term cost $J(\boldsymbol{\theta}) = \sum_t \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]$

## PILCO Framework: High-Level Steps

1. Probabilistic model for transition function $f$
   ▶▶ System identification

Deisenroth et al. (IEEE-TPAMI, 2015): *Gaussian Processes for Data-Efficient Learning in Robotics and Control*

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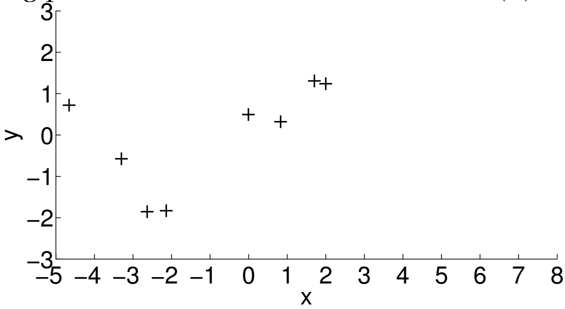
# Model Learning (System Identification)

Model learning problem: Find a function $f : x \mapsto f(x) = y$




Observed function values

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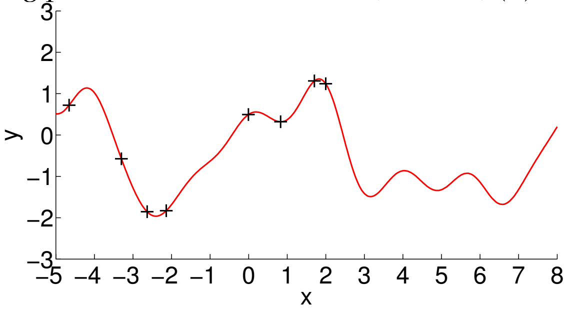


Plausible model

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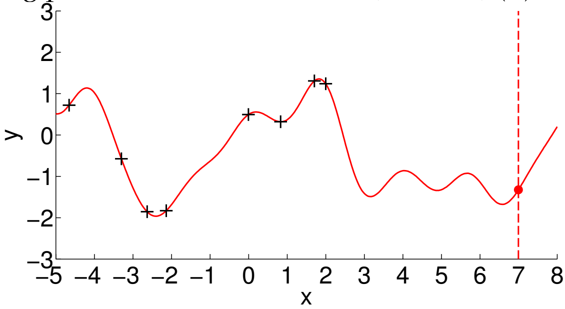


Plausible model

**Predictions? Decision Making?**

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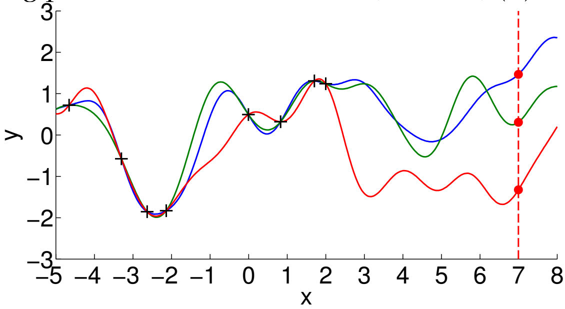
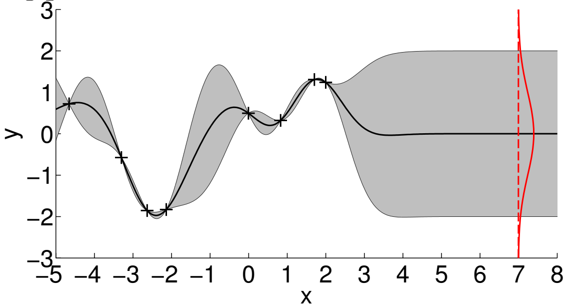



More plausible models

**Predictions? Decision Making? Model Errors!**

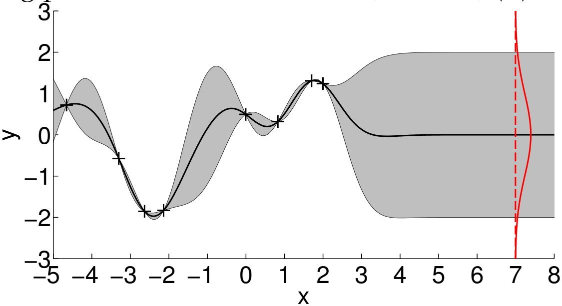
# Model Learning (System Identification)

Model learning problem: Find a function $f : x \mapsto f(x) = y$




Distribution over plausible functions

Model learning problem: Find a function $f : x \mapsto f(x) = y$




Distribution over plausible functions

- Express uncertainty about the underlying function to be robust to model errors
- Gaussian process for model learning (Rasmussen & Williams, 2006)

## Objective

Minimize expected long-term cost $J(\boldsymbol{\theta}) = \sum_t \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]$

### PILCO Framework: High-Level Steps

1. Probabilistic model for transition function $f$
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2. **Compute long-term predictions** $p(\boldsymbol{x}_1|\boldsymbol{\theta}), \dots, p(\boldsymbol{x}_T|\boldsymbol{\theta})$
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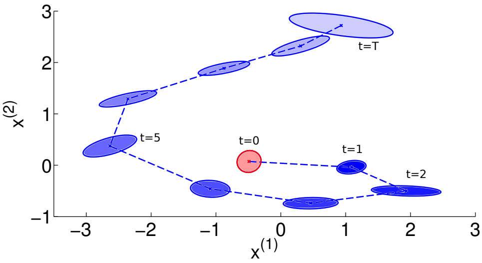
Deisenroth et al. (IEEE-TPAMI, 2015): *Gaussian Processes for Data-Efficient Learning in Robotics and Control*

# Long-Term Predictions




- Iteratively compute $p(\boldsymbol{x}_1|\boldsymbol{\theta}), \ldots, p(\boldsymbol{x}_T|\boldsymbol{\theta})$

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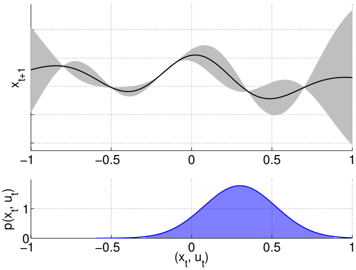


- Iteratively compute $p(\boldsymbol{x}_1|\boldsymbol{\theta}), \ldots, p(\boldsymbol{x}_T|\boldsymbol{\theta})$

$$\underbrace{p(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t)}_{\text{GP prediction}} \; \underbrace{p(\boldsymbol{x}_t, \boldsymbol{u}_t|\boldsymbol{\theta})}_{\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})}$$

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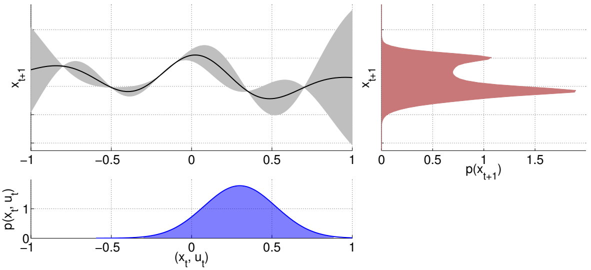
# Long-Term Predictions

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$$p(\boldsymbol{x}_{t+1}|\boldsymbol{\theta}) = \iiint \underbrace{p(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t)}_{\text{GP prediction}} \underbrace{p(\boldsymbol{x}_t, \boldsymbol{u}_t|\boldsymbol{\theta})}_{\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})} df \, d\boldsymbol{x}_t \, d\boldsymbol{u}_t$$

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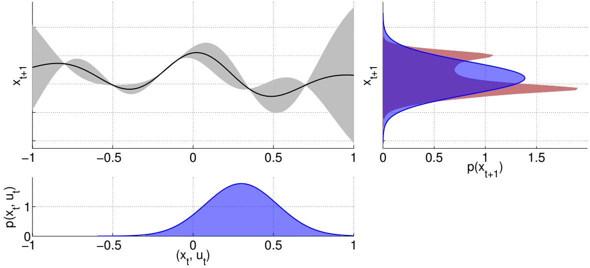
# Long-Term Predictions

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▶▶ GP moment matching
(Girard et al., 2002; Quiñonero-Candela et al., 2003)

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   - Compute expected long-term cost $J(\boldsymbol{\theta})$
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# Policy Improvement

## Objective

Minimize expected long-term cost $J(\boldsymbol{\theta}) = \sum_t \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]$

- Know how to predict $p(\boldsymbol{x}_1|\boldsymbol{\theta}), \ldots, p(\boldsymbol{x}_T|\boldsymbol{\theta})$

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- Compute

$$\mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}] = \int c(\boldsymbol{x}_t)\mathcal{N}\big(\boldsymbol{x}_t \,|\, \boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t\big) d\boldsymbol{x}_t\,, \quad t = 1, \dots, T\,,$$

and sum them up to obtain $J(\boldsymbol{\theta})$

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# Policy Improvement

UCL

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  and sum them up to obtain $J(\boldsymbol{\theta})$
- Analytically compute gradient $\mathrm{d}J(\boldsymbol{\theta})/\mathrm{d}\boldsymbol{\theta}$
- Standard gradient-based optimizer (e.g., BFGS) to find $\boldsymbol{\theta}^*$

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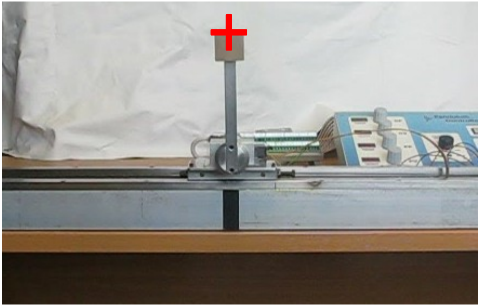
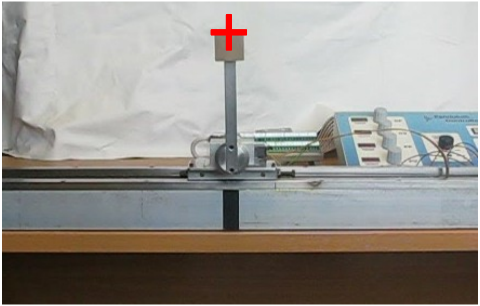
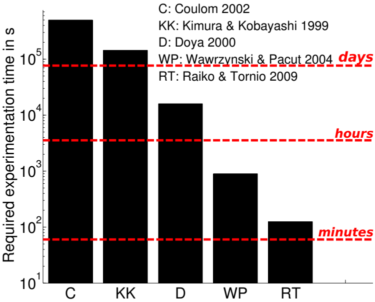


- Swing up and balance a freely swinging pendulum on a cart
- No knowledge about nonlinear dynamics ⏩ Learn from scratch
- Cost function $c(\boldsymbol{x}) = 1 - \exp(-\|\boldsymbol{x} - \boldsymbol{x}_{\text{target}}\|^2)$
- Code: https://github.com/ICL-SML/pilco-matlab

Deisenroth & Rasmussen (ICML, 2011): *PILCO: A Model-based and Data-efficient Approach to Policy Search*

# Standard Benchmark: Cart-Pole Swing-up



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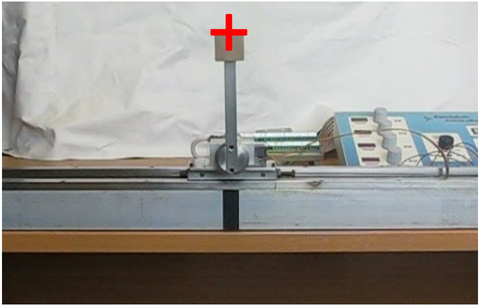

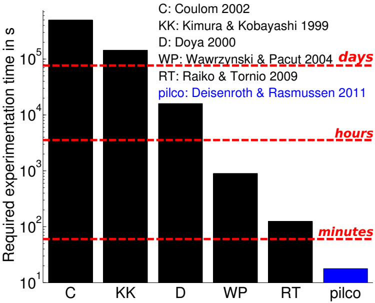


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- No knowledge about nonlinear dynamics ➤➤ Learn from scratch
- Cost function $c(\boldsymbol{x}) = 1 - \exp(-\|\boldsymbol{x} - \boldsymbol{x}_{\text{target}}\|^2)$
- **Unprecedented learning speed** compared to state-of-the-art
- Code: `https://github.com/ICL-SML/pilco-matlab`

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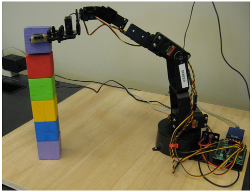
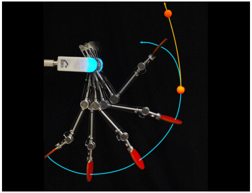
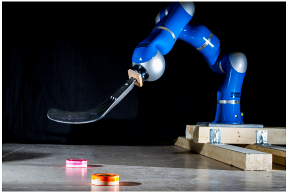
# Wide Applicability



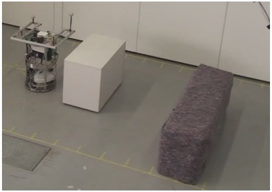
with D Fox



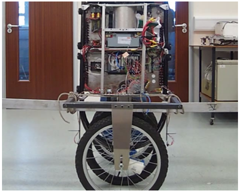
with P Englert, A Paraschos, J Peters



with A Kupcsik, J Peters, G Neumann



B Bischoff (Bosch), ESANN 2013

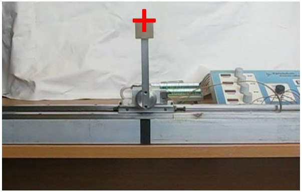


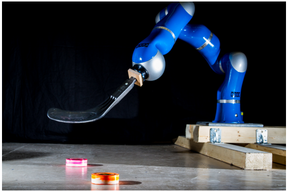
A McHutchon (U Cambridge)

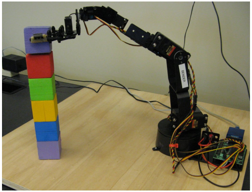
▶▶ Application to a wide range of robotic systems


Deisenroth et al. (RSS, 2011): *Learning to Control a Low-Cost Manipulator using Data-efficient Reinforcement Learning*
Englert et al. (ICRA, 2013): *Model-based Imitation Learning by Probabilistic Trajectory Matching*
Deisenroth et al. (ICRA, 2014): *Multi-Task Policy Search for Robotics*
Kupcsik et al. (AIJ, 2017): *Model-based Contextual Policy Search for Data-Efficient Generalization of Robot Skills*

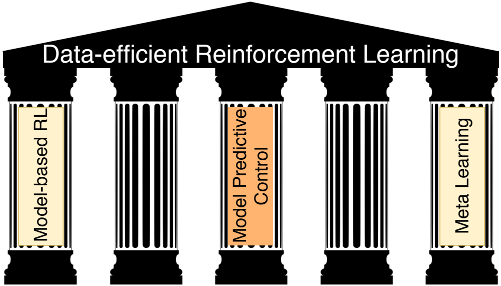
# Summary (1)



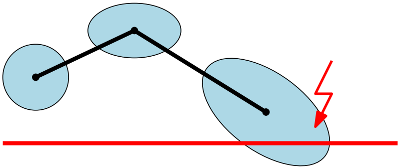




- In robotics, data-efficient learning is critical
- Probabilistic, model-based RL approach
  - Reduce model bias
  - Unprecedented learning speed
  - Wide applicability


Data-efficient Reinforcement Learning
Model-based RL
Model Predictive Control
Meta Learning

# Safe Exploration



- Deal with real-world safety constraints
- Use probabilistic model to predict whether constraints are violated (e.g., Sui et al., 2015; Berkenkamp et al., 2017)
- Adjust policy if necessary (during policy learning)

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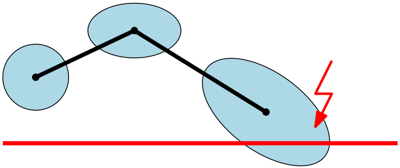

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▶▶ Safe exploration within an MPC-based RL setting
▶▶ Optimize control signals $\boldsymbol{u}_t$ directly (no policy parameters)

# Approach

- Idea: Optimize control signals directly (instead of policy parameters)
- Few parameters to optimize ⏩ Low-dimensional search space
- Open-loop control
  ⏩ No chance of success (with minor model inaccuracies)

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- Few parameters to optimize ⏩ Low-dimensional search space
- Open-loop control
  ⏩ No chance of success (with minor model inaccuracies)
- Model Predictive Control (MPC) turns this into a closed-loop control approach
- Positive side-effect: Increase robustness to model errors (online approach)
  ⏩ Increase data efficiency

# Probabilistic MPC in RL

- GP model for transition dynamics
- Repeat (while executing the policy):
  1. In current state $\boldsymbol{x}_t$, determine optimal control sequence $\boldsymbol{u}_1^*, \ldots, \boldsymbol{u}_H^*$
  2. Apply first control $\boldsymbol{u}_1^*$ in state $\boldsymbol{x}_t$
  3. Transition to next state $\boldsymbol{x}_{t+1}$
  4. Update GP transition model

Kamthe & Deisenroth (AISTATS, 2018): *Data-Efficient Reinforcement Learning with Probabilistic Model Predictive Control*

# Theoretical Results

- Uncertainty propagation is deterministic (GP moment matching)
  - Re-formulate system dynamics:

$$z_t := \{\mu_t, \Sigma_t\} \quad \text{Collects moments}$$
$$z_{t+1} = f_{MM}(z_t, u_t)$$

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 ▸▸ Principled treatment of control constraints

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  ▸▸ Principled treatment of control constraints
- Use predictive uncertainty to check whether state constraints are violated

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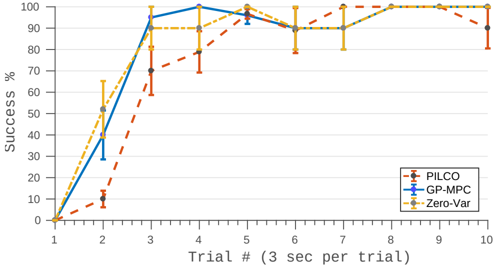
# Experimental Results: Learning Speed




- Zero-Var: Only use the mean of the GP, discard variances for long-term predictions
- MPC: Increased data efficiency (40% less experience required than PILCO)
  ▶▶ MPC is more robust to model inaccuracies than a parametrized feedback controller

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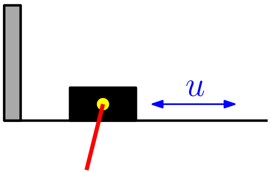
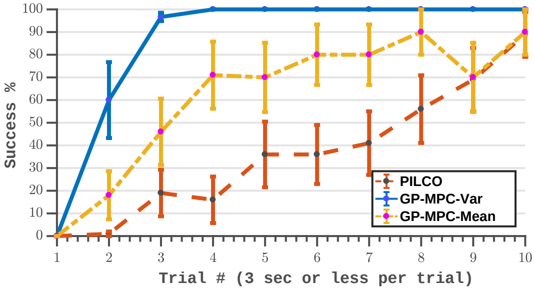
# Experimental Results: Safety Constraints



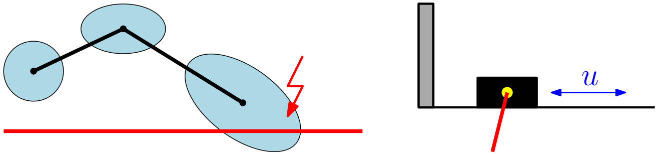




| | | |
|---|---|---|
| PILCO | 16/100 | constraint violations |
| GP-MPC-Mean | 21/100 | constraint violations |
| GP-MPC-Var | 3/100 | constraint violations |

**Propagating model uncertainty important for safety**

Kamthe & Deisenroth (AISTATS, 2018): *Data-Efficient Reinforcement Learning with Probabilistic Model Predictive Control*

# Summary (2)




- Probabilistic prediction models for safe exploration
- Uncertainty propagation can be used to reduce violation of safety constraints
- MPC framework increases robustness to model errors
  ⏩ Increased data efficiency


Data-efficient Reinforcement Learning
Model-based RL
Model Predictive Control
Meta Learning

# Meta Learning




## Meta Learning

Generalize knowledge from known tasks to new (related) tasks

- Different robot configurations (link lengths, weights, ...)
- Re-use experience gathered so far generalize learning to new dynamics that are similar
  ▶▶ Accelerated learning

# Approach

- Separate global and task-specific properties
- Shared global parameters describe general dynamics
- Describe task-specific (local) configurations with latent variable

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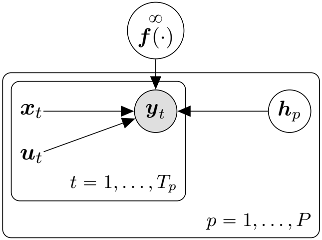
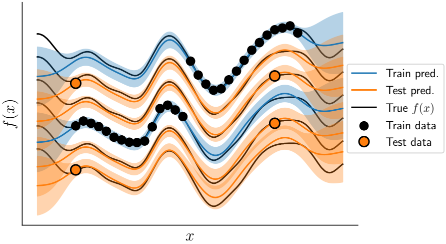
- Separate global and task-specific properties
- Shared global parameters describe general dynamics
- Describe task-specific (local) configurations with latent variable
- Online variational inference of (unseen) configurations
- Few-shot model-based RL

# Meta Model Learning with Latent Variables



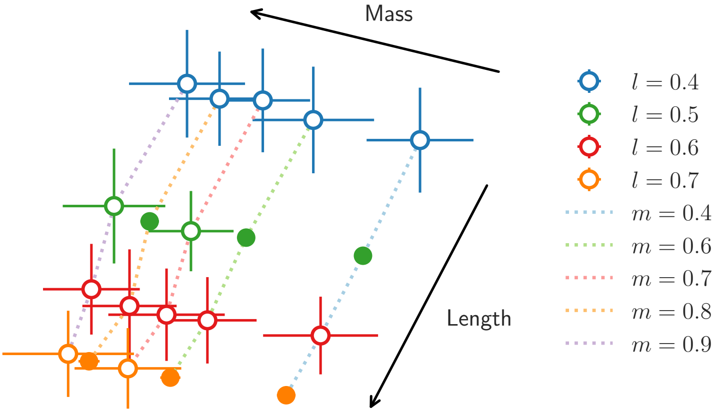




$$\boldsymbol{y}_t = \boldsymbol{f}(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{h}_p)$$

- GP captures global properties of the dynamics
- Latent variable $\boldsymbol{h}_p$ describes local configuration
  ▸▸ Variational inference to find a posterior on latent configuration
- Fast online inference of new configurations (no model re-training required)

Sæmundsson et al. (UAI, 2018): *Meta Reinforcement Learning with Latent Variable Gaussian Processes*

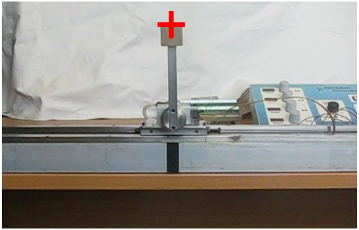
# Latent Embeddings




- Latent variable $\boldsymbol{h}$ encodes length $l$ and mass $m$ of the cart pole
- 6 training tasks, 14 held-out test tasks

Sæmundsson et al. (UAI, 2018): *Meta Reinforcement Learning with Latent Variable Gaussian Processes*

# Meta-RL (Cart Pole): Training



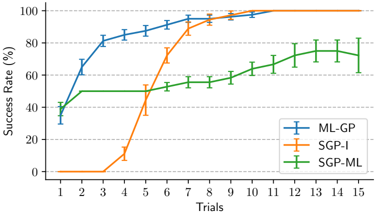
- Pre-trained on 6 training configurations until solved

| Model | Training (s) | Description |
|---|---|---|
| Independent | $16.1 \pm 0.4$ | Independent GP-MPC |
| Aggregated | $23.7 \pm 1.4$ | Aggregated experience (no latents) |
| Meta learning | $\mathbf{15.1 \pm 0.5}$ | Aggregated experience (with latents) |

▶▶ **Meta learning can help speeding up RL**

Sæmundsson et al. (UAI, 2018): *Meta Reinforcement Learning with Latent Variable Gaussian Processes*

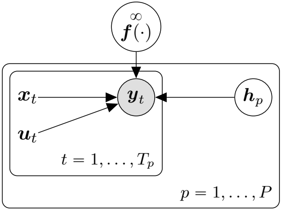
# Meta-RL (Cart Pole): Few-Shot Generalization

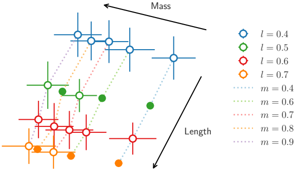



- Few-shot generalization on 4 unseen configurations
- Success: solve all 10 (6 training + 4 test) tasks
- Meta learning: blue
- Independent (GP-MPC): orange
- Aggregated experience model (no latents): green

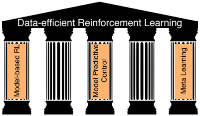
▸▸ **Meta RL generalizes well to unseen tasks**

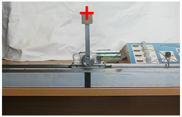
Sæmundsson et al. (UAI, 2018): *Meta Reinforcement Learning with Latent Variable Gaussian Processes*

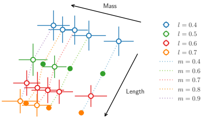



- Generalize knowledge from known situations to unseen ones
  ▶▶ **Few-shot learning**
- Latent variable can be used to describe how related tasks are
- Significant speed-up in model learning and model-based RL

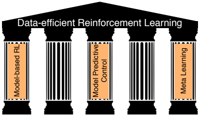


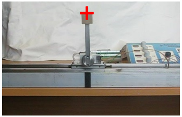



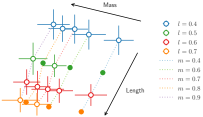



- **Data efficiency** is a practical challenge for autonomous robots
- Three pillars of data-efficient reinforcement learning
    1. Probabilistic model-based RL for fast learning of models and controllers
    2. Model predictive control with learned dynamics models accelerate learning and allow for safe exploration
    3. Meta learning using latent variables to generalize knowledge to new situations

# Wrap-Up



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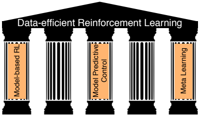


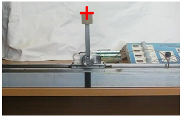

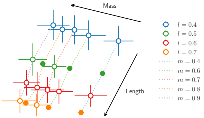


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**Thank you for your attention**

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# GP Moment Matching: Some Details

$$f \sim GP(0, k)\,, \quad \text{Training data: } \boldsymbol{X}, \boldsymbol{y}$$
$$\boldsymbol{x}_* \sim \mathcal{N}\big(\boldsymbol{\mu},\, \boldsymbol{\Sigma}\big)$$

- Compute $\mathbb{E}[f(\boldsymbol{x}_*)]$

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▶▶ independent of $\boldsymbol{x}_*$

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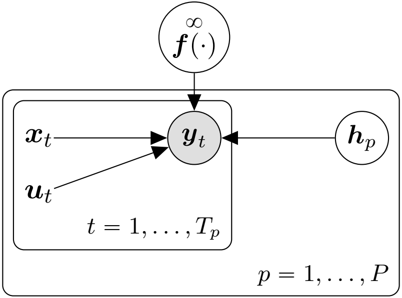
$$\boldsymbol{\beta} := (\boldsymbol{K} + \sigma_n^2\boldsymbol{I})^{-1}\boldsymbol{y}$$

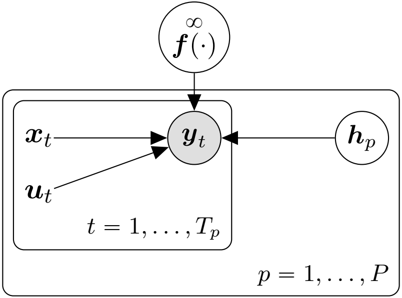
➡ independent of $\boldsymbol{x}_*$

- If $k$ is a Gaussian (squared exponential) kernel, this integral can be solved analytically
- Variance of $f(\boldsymbol{x}_*)$ can be computed similarly




$$\boldsymbol{f}(\cdot) \sim GP$$

$$p(\boldsymbol{H}) = \prod_p p(\boldsymbol{h}_p)\,, \quad p(\boldsymbol{h}_p) = \mathcal{N}\big(\boldsymbol{0},\, \boldsymbol{I}\big)$$

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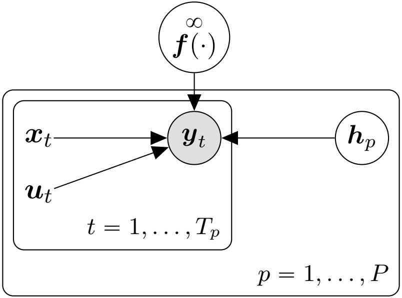
$$p(\boldsymbol{Y}, \boldsymbol{H}, \boldsymbol{f}(\cdot) | \boldsymbol{X}, \boldsymbol{U}) = \prod_{p=1}^{P} p(\boldsymbol{h}_p) \prod_{t=1}^{T_p} p(\boldsymbol{y}_t | \boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{h}_p, \boldsymbol{f}(\cdot)) p(\boldsymbol{f}(\cdot))$$

$$\boldsymbol{y}_t = \boldsymbol{x}_{t+1} - \boldsymbol{x}_t$$




Mean-field variational family:

$$q(\boldsymbol{f}(\cdot), \boldsymbol{H}) = q(\boldsymbol{f}(\cdot))q(\boldsymbol{H})$$

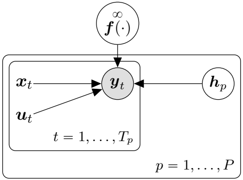
$$q(\boldsymbol{H}) = \prod_{p=1}^{P} \mathcal{N}(\boldsymbol{h}_p | \boldsymbol{n}_p, \boldsymbol{T}_p) \,,$$

$$q(\boldsymbol{f}(\cdot)) = \int p(\boldsymbol{f}(\cdot)|\boldsymbol{f}_Z)q(\boldsymbol{f}_Z)d\boldsymbol{f}_Z$$

▶▶ SV-GP (Titsias, 2009)

# Evidence Lower Bound

$$ELBO = \mathbb{E}_{q(\boldsymbol{f}(\cdot),\boldsymbol{H})}\Big[\log \frac{p(\boldsymbol{Y},\boldsymbol{H},\boldsymbol{f}(\cdot)|\boldsymbol{X},\boldsymbol{U})}{q(\boldsymbol{f}(\cdot),\boldsymbol{H})}\Big]$$



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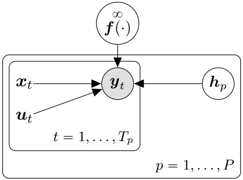
$$= \sum_{p=1}^{P}\sum_{t=1}^{T_p}\mathbb{E}_{q(\boldsymbol{f}_t|\boldsymbol{x}_t,\boldsymbol{u}_t,\boldsymbol{h}_p)q(\boldsymbol{h}_p)}\big[\log p(\boldsymbol{y}_t|\boldsymbol{f}_t)\big]$$

$$- \mathrm{KL}(q(\boldsymbol{H})||p(\boldsymbol{H})) - \mathrm{KL}(q(\boldsymbol{f}(\cdot))||p(\boldsymbol{f}(\cdot)))$$



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$$= \sum_{p=1}^{P}\sum_{t=1}^{T_p}\overbrace{\mathbb{E}_{q(\boldsymbol{f}_t|\boldsymbol{x}_t,\boldsymbol{u}_t,\boldsymbol{h}_p)q(\boldsymbol{h}_p)}\big[\log p(\boldsymbol{y}_t|\boldsymbol{f}_t)\big]}^{\text{Monte Carlo estimate}} \underbrace{- \mathrm{KL}(q(\boldsymbol{H})||p(\boldsymbol{H})) - \mathrm{KL}(q(\boldsymbol{F}_Z)||p(\boldsymbol{F}_Z))}_{\text{closed-form solution}}$$