PILCO: A Model-Based and Data-Efficient Approach to Policy Search

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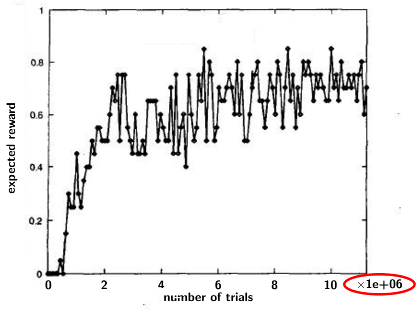
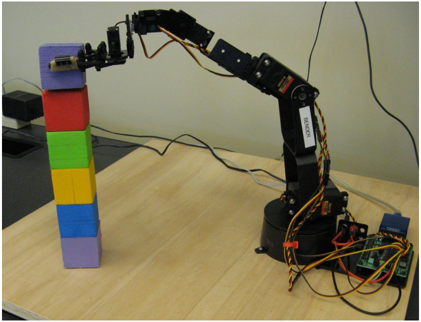
Motivation



(a) Typical learning curve for cart-pole balancing.



(b) Lynxmotion robotic arm.

- RL often data inefficient if we learn from scratch: needs too many trials
  ➡ largely inapplicable to mechanical systems
- Make RL **more data efficient** (get away with fewer trials)
  - More informative prior knowledge (e.g., demonstrations, system equations)
  - **Extract more valuable information from data**

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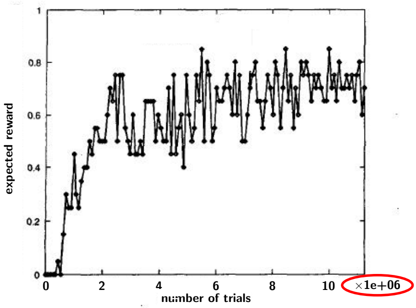
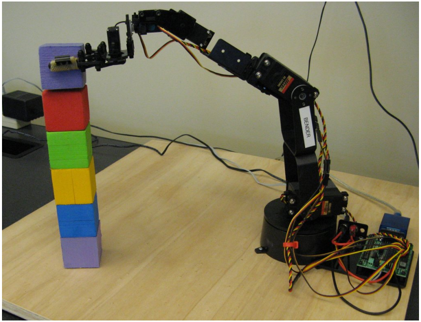


(c) Typical learning curve for cart-pole balancing.



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Problem Formulation

Objective

Learn a **policy** π^* that yields minimal **expected long-term cost** $J^\pi(\boldsymbol{\theta})$

$$J^\pi(\boldsymbol{\theta}) = \sum_{t=0}^{T} \mathbb{E}_{\mathbf{x}_t}[c(\mathbf{x}_t)|\pi]$$

Follow π for T steps starting from $p(\mathbf{x}_0)$

- **Policy parameters** $\boldsymbol{\theta}$
- **Cost** $c(\mathbf{x}_t)$ of being in state $\mathbf{x}_t$. We choose

$$c(\mathbf{x}_t) = 1 - \exp(-\tfrac{1}{2}\|\mathbf{x}_t - \mathbf{x}_{\text{target}}\|^2/\sigma_c^2) \quad \in [0, 1]$$

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Challenges:

- **Data-efficient** solution (few trials)
- **Unknown transition dynamics** $f : (\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) \mapsto \mathbf{x}_t$
- **No expert knowledge/demonstrations** available ➡ learn from scratch

Making RL Efficient

Model-based RL

- Learn model of transition dynamics f
- Use model for internal simulation ➡ certainty equivalence assumption (Schneider, NIPS 1997; Bagnell and Schneider, ICRA 2001)
- Learn policy based on these simulations
- Hope: few interactions with system
  ➡ suffers from **model errors**, but can be **data efficient**

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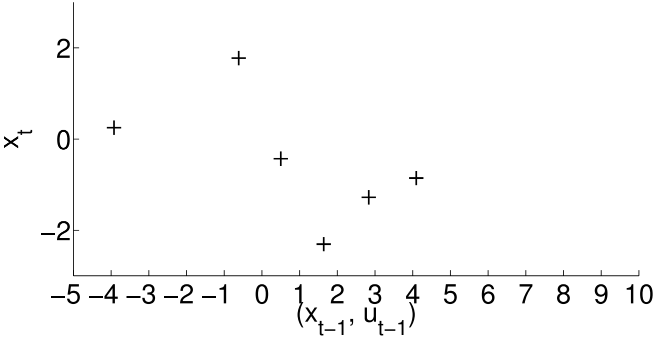
➡ Being efficient (often) requires **dealing with model errors** (Atkeson and Santamaría, ICML 1997)

Dealing with Model Errors

Task: find a (transition) function $f : (\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) \mapsto \mathbf{x}_t$



Training set for model learning

Dealing with Model Errors

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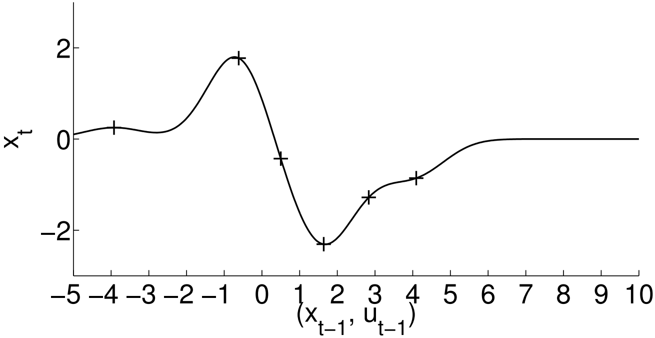

Deterministic (MAP) function approximator

Dealing with Model Errors

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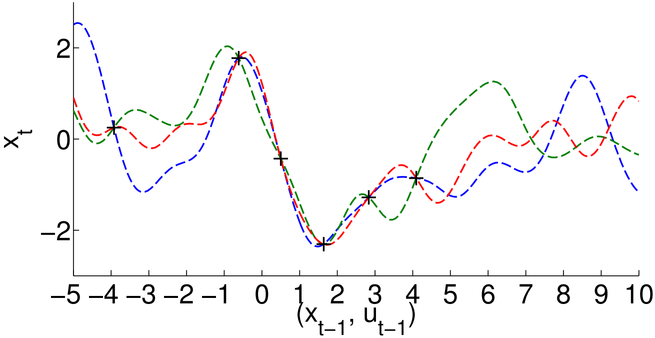

Other plausible function approximators

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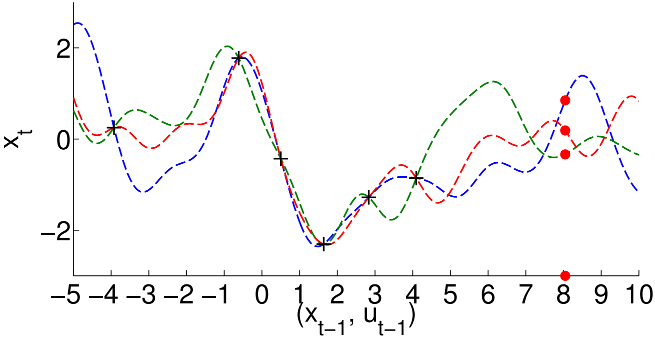

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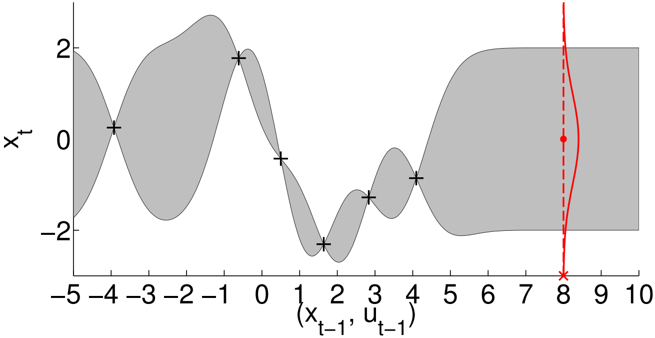

Probabilistic function approximator: distribution over plausible functions

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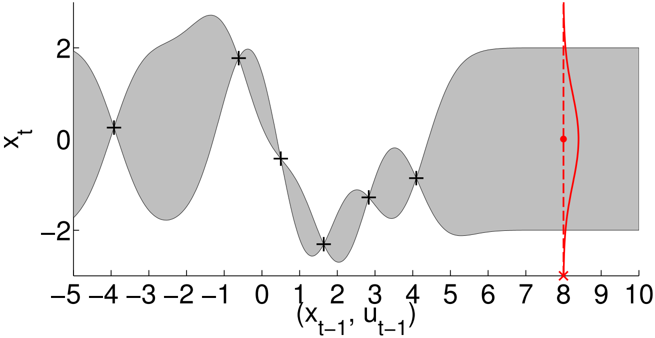

Probabilistic function approximator: distribution over plausible functions

- Express **model uncertainty** about the function at unobserved locations
- **Must** use a **probabilistic** function approximator
- **Pilco** framework (Nonparametric Gaussian processes for dynamics model)

PILCO

- Probabilistic inference for learning control
- Model-based **policy search** method with **analytic policy gradients**
  ➡ find good policy parameters $\boldsymbol{\theta}^*$
- **Gaussian processes** for probabilistic dynamics model
  zero prior mean, SE covariance function

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  ➡ Take them into account during planning
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  ➡ Allows for learning from scratch (episodic tasks)

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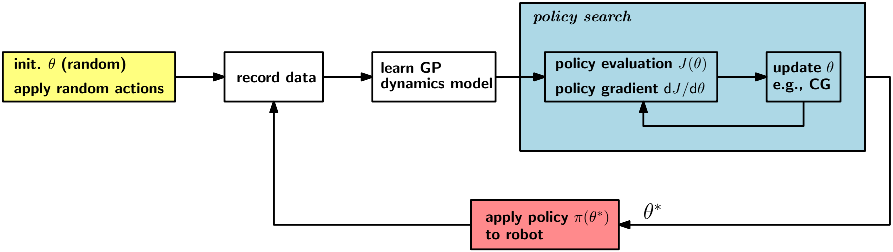
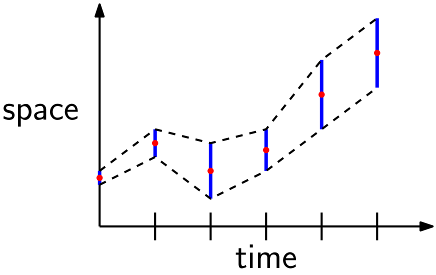


Approximate Inference for Policy Evaluation

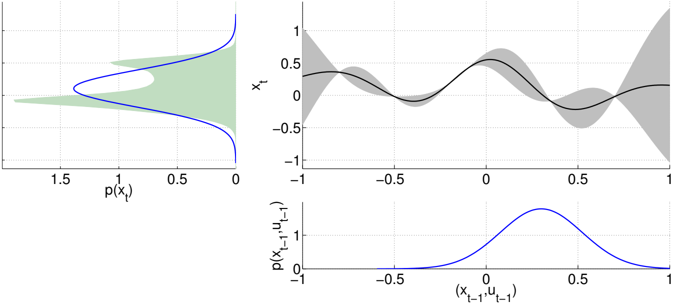
- Want to compute $J^{\pi}(\boldsymbol{\theta}) = \sum_t \mathbb{E}[c(\mathbf{x}_t)]$
- Obtain one-step transition probabilities $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$ from GP dynamics model
- Idea: **cascade** predictions to get $p(\mathbf{x}_1), \dots, p(\mathbf{x}_T)$



➡ $J^{\pi}(\boldsymbol{\theta})$ can be evaluated (assuming $\mathbb{E}_{\mathbf{x}}[c(\mathbf{x})]$ can be computed)

Approximate Inference for Policy Evaluation (2)

- **Problem:** predictions $p(\mathbf{x}_1), \ldots, p(\mathbf{x}_T)$ cannot be computed exactly.
- Approximate inference required
  - Robust **moment matching** approximation of predictive distribution (Quiñonero-Candela et al., ICASSP 2003; Deisenroth et al., ICML 2009)



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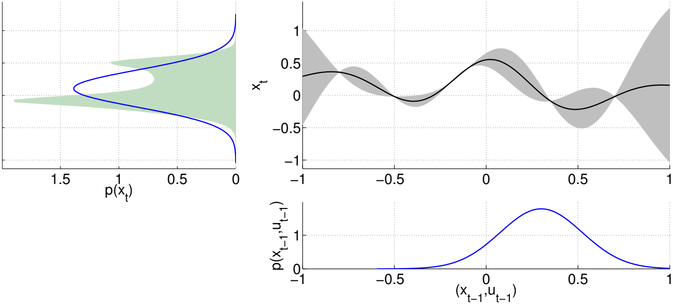
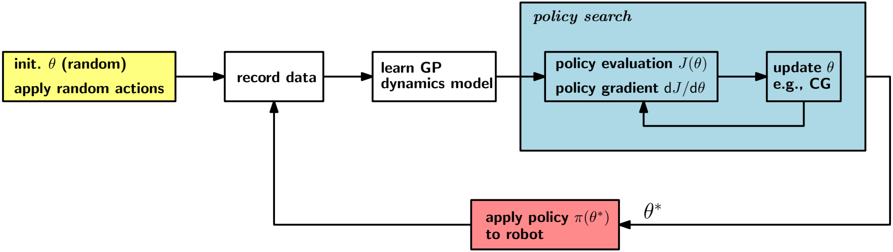


➡ Get approximate Gaussian state distributions $p(\mathbf{x}_1), \dots, p(\mathbf{x}_T)$

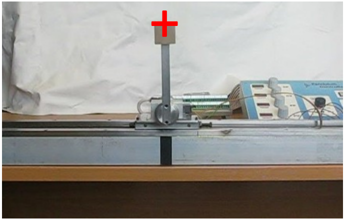
➡ **Analytic policy evaluation and policy gradients d$J^\pi(\boldsymbol{\theta})$/d$\boldsymbol{\theta}$**

Results

- Hardware applicability
- High-dimensional problems
- Data efficiency

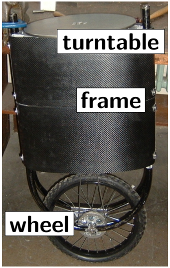


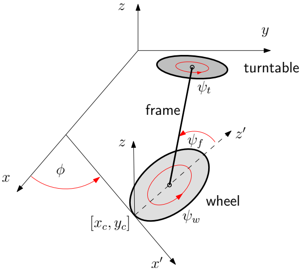
Standard Benchmark Problem

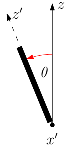


- State space: $\mathbf{x} \in \mathbb{R}^4$
- Policy parameters: $\boldsymbol{\theta} \in \mathbb{R}^{300}$
- Control frequency: 10 Hz
- $<$ 10 trials
- $\approx$ 20 seconds of interaction time

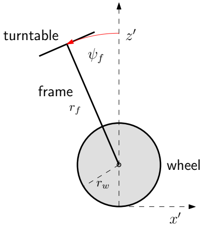
Scaling to Higher Dimensions: Unicycling







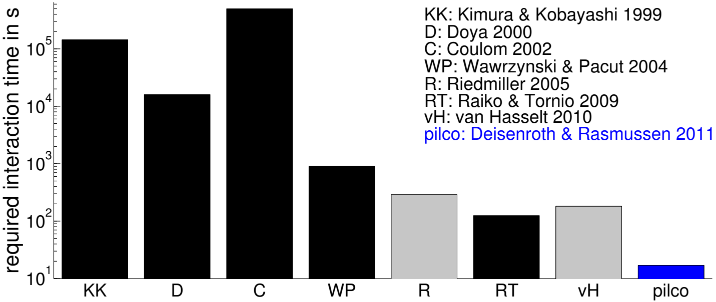
sideways tilt of wheel



tilt of the frame

- State space: $\mathbf{x} \in \mathbb{R}^{12}$, $\boldsymbol{\theta} \in \mathbb{R}^{26}$
- 2-dimensional controls (wheel torque and flywheel torque)
- Control frequency: 6.66 Hz
- $\approx$ 15–20 trials (including 5 random trials)
- $\approx$ 30 seconds interaction time

Data Efficiency



Cart-pole task (results from literature)

- Only "**learning from scratch**" (no demonstrations etc.)
- Gray bars: balancing
- Black bars: swing up and balancing
- Slightly different setups (masses, rewards, discretization)
- About one **order of magnitude less interaction** time than best other method

Wrap-up

- PILCO: Data-efficient model-based policy search method
- **No expert knowledge/demonstrations** required
- Key point: **reduce model errors** by using probabilistic dynamics models
- **Unprecedented speed of learning**
- Hardware applicability, scaling to high dimensions

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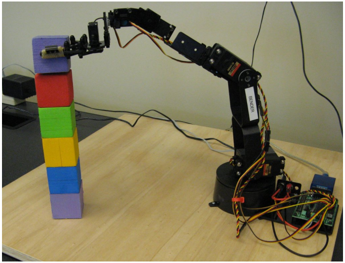
`http://mlg.eng.cam.ac.uk/carl/pilco`

`http://www.cs.washington.edu/homes/marc/pilco`

`marc@cs.washington.edu`

Controlling a Really Noisy Robot



- Low-cost robotic manipulator
- Kinect-style depth camera only sensor
- Learn to stack blocks (from scratch)

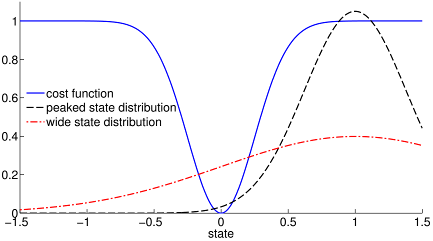
(Deisenroth et al., R:SS 2011)

Parameters to be set

- number of basis functions (policy)
- general system properties (e.g. length of pendulum)
- cost function
- control frequency (Δ_t)
- length of control/prediction horizon T

Exploration/Exploitation

- Compute $\mathbb{E}[c(\mathbf{x}_t)]$
- We choose $c(\mathbf{x}) = 1 - \exp(-\frac{1}{2}\|\mathbf{x} - \mathbf{x}_{\text{target}}\|^2/\sigma_c^2)$



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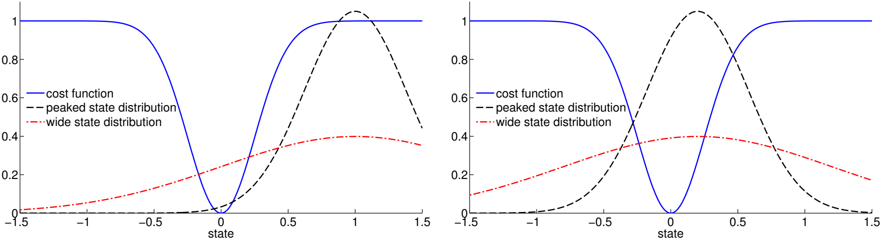

- Far away from the target, uncertainty (this comes from averaging out model uncertainty!) is favorable ➡ **explore**
- Close to the target, we want to be certain ➡ **exploit**

Computational complexity

- training dynamics models

$$\mathcal{O}(dn^3)$$

- predictions (policy evaluation)

$$\mathcal{O}(d^3n^2)$$

➡ sparse approximations speed up (factor n)

Policy improvement

- policy: parameterized function (parameters $\boldsymbol{\theta}$)
- $\mathbf{x}_t$ is a function of $\boldsymbol{\theta}$ through

$$\begin{aligned} \mathbf{x}_t &= f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})\,, \\ \mathbf{u}_{t-1} &= \pi(\mathbf{x}_{t-1}, \boldsymbol{\theta}) \end{aligned}$$

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- policy evaluation can be done analytically (with approximations)
  ➡ **analytic gradients** (chain rule) are available:

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$$\frac{\mathrm{d}J^{\pi}(\boldsymbol{\theta})}{\mathrm{d}\boldsymbol{\theta}}$$

➡ use your favorite toolbox for nonconvex optimization to get $\boldsymbol{\theta}^*$

➡ **no value function model** required

Policy parametrization

$$\pi(\mathbf{x}) = \sum_{i=1}^{n} w_i \phi_i(\mathbf{x}) = \sum_{i=1}^{n} w_i \exp\left(-\tfrac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_i)^\top \boldsymbol{\Lambda}^{-1}(\mathbf{x}-\boldsymbol{\mu}_i)\right)$$

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policy parameters $\boldsymbol{\theta}$

- n weights w_i (per control dimension)
- d length-scales $\ell_1, \dots, \ell_d$ (per control dimension)
- n centers $\boldsymbol{\mu}_i \in \mathbb{R}^d$ of basis functions (shared across control dimensions)

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- n centers $\boldsymbol{\mu}_i \in \mathbb{R}^d$ of basis functions (shared across control dimensions)
- $(d+1)n + d$ parameters
  example: $n = 50, d = 6, \mathsf{dim}(\mathbf{u}) = 2 \longrightarrow |\boldsymbol{\theta}| \approx 400$

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